INTRODUCTION

Where "Numbers" Come From

TEACHER LEARNS A LESSON

This book was sparked off when I was a schoolteacher by questions asked by children. Like any decent teacher, I tried not to leave any question unanswered, however odd or naive it might seem. After all, a curious mind often is an intelligent one.

One morning, I was giving a class about the way we write down numbers. I had done my own homework and was well-prepared to explain the ins and outs of the splendid system that we have for representing numbers in Arabic numerals, and to use the story to show the theoretical possibility of shifting from base 10 to any other base without altering the properties of the numbers or the nature of the operations that we can carry out on them. In other words, a perfectly ordinary maths lesson, the sort of lesson you might have once sat through yourself – a lesson taught, year in, year out, since the very foundation of secondary schooling.

But it did not turn out to be an ordinary class. Fate, or Innocence, made that day quite special for me.

Some pupils – the sort you would not like to come across too often, for they can change your whole life! – asked me point-blank all the questions that children have been storing up for centuries. They were such simple questions that they left me speechless for a moment:

"Sir, where do numbers come from? Who invented zero?"

Well, where do numbers come from, in fact? These familiar symbols seem so utterly obvious to us that we have the quite mistaken impression that they sprang forth fully formed, as gods or heroes are supposed to. The question was disconcerting. I confess I had never previously wondered what the answer might be.

"They come . . . er . . . they come from the remotest past," I fumbled, barely masking my ignorance.

But I only had to think of Latin numbering (those Roman numerals which we still use to indicate particular kinds of numbers, like sequences of kings or millionaires of the same name) to be quite sure that numbers have not always been written in the same way as they are now.

"Sir!" said another boy, "Can you tell us how the Romans did their sums? I've been trying to do a multiplication with Roman numerals for days, and I'm getting nowhere with it!"

"You can't do sums with those numerals," another boy butted in. "My dad told me the Romans did their sums like the Chinese do today, with an abacus."

That was almost the right answer, but one which I didn't even possess.

"Anyway," said the boy to the rest of the class, "if you just go into a Chinese restaurant you'll see that those people don't need numbers or calculators to do their sums as fast as we do. With their abacuses, they can even go thousands of times faster than the biggest computer in the world."

That was a slight exaggeration, though it is certainly true that skilled abacists can make calculations faster than they can be done on paper or on mechanical calculating machines. But modern electronic computers and calculators obviously leave the abacus standing.

I was fortunate and privileged to have a class of boys from very varied backgrounds. I learned a lot from them.

"My father's an ethnologist," said one. "He told me that in Africa and Australia there are still primitive people so stupid that they can't even count further than two! They're still cavemen!"

What extraordinary injustice in the mouth of a child! Unfortunately, there used to be plenty of so-called experts who believed, as he did, that "primitive" peoples had remained at the first stages of human evolution. However, when you look more closely, it becomes apparent that "savages" aren't so stupid after all, that they are far from being devoid of intelligence, and that they have extraordinarily clever ways of coping without numbers. They have the same potential as we all do, but their cultures are just very different from those of "civilised" societies.

But I did not know any of that at the time. I tried to grope my way back through the centuries. Before Arabic numerals, there were Roman ones. But does "before" actually mean anything? And even if it did, what was there before those numerals? Was it going to be possible to use an archaeology of numerals and computation to track back to that mind-boggling moment when someone first came up with the idea of counting?

Several other allegedly naive questions arose as a result of my pupils' curious minds. Some concerned "counting animals" that you sometimes see at circuses and fairs; they are supposed to be able to count (which is why some people claim that mathematicians are just circus artistes!) Other pupils put forward the puzzle of "number 13", alternately considered an omen of good luck and an omen of bad luck. Others wondered what was in the minds of mathematical prodigies, those phenomenal beings who can perform very complex operations in their heads at high speed – calculating the cube root of a fifteen-digit number, or reeling off all the prime numbers between seven million and ten million, and so on.

In a word, a whole host of horrendous but fascinating questions exploded in the face of a teacher who, on the verge of humiliation, took the full measure of his ignorance and began to see just how inadequate the teaching of mathematics is if it makes no reference to the history of the subject. The only answers I could give were improvised ones, incomplete and certainly incorrect.

I had an excuse, all the same. The arithmetic books and the school manuals which were my working tools did not even allude to the history of numbers. History textbooks talk of Hammurabi, Caesar, King Arthur, and Charlemagne, just as they mention the travels of Marco Polo and Christopher Columbus; they deal with topics as varied as the history of paper, printing, steam power, coinage, economics, and the calendar, as well as the history of human languages and the origins of writing and of the alphabet. But I searched them in vain for the slightest mention of the history of numbers. It was almost as if a conspiracy of obviousness aimed to make a secret, or, even worse, just to make us ignorant of one of the most fantastic and fertile of human discoveries. Counting is what allowed people to take the measure of their world, to understand it better, and to put some of its innumerable secrets to good use.

These questions had a profound impact on me, beginning with this lesson in modesty: my pupils, who were manifestly more inquisitive than I had been, taught me a lesson by spurring me on to study the history of a great invention. It turned out to be a history that I quickly discovered to be both universal and discontinuous.

THE QUEST FOR THE MATHEMATICAL GRAIL

I could not now ever let go of these questions, and they soon drew me into the most fascinating period of learning and the most enthralling adventure of my life.

My desire to find the answers and to have time to think about them persuaded me, not without regrets, to give up my teaching job. Though I had only slender means, I devoted myself full-time to a research project that must have seemed as mad, in the eyes of many people, as the mediaeval quest for the Holy Grail, the magical vessel in which the blood of Christ on the cross was supposed to have been collected. Lancelot, Perceval, and Gawain, amongst many other valiant knights of Christendom, set off in search of the grail without ever completing their quest, because they were not pure enough or lacked sufficient faith or chastity to approach the Truth of God.

I couldn't claim to have chastity or purity either. But faith and calling led me to cross the five continents, materially or intellectually, and to glimpse horizons far wider than those that the cloistered world of mathematics usually allows. But the more my eyes opened onto the wider world, the more I realised the depth of my ignorance.

Where, when and how did the amazing adventure of the human intellect begin? In Asia? In Europe? Or somewhere in Africa? Did it take place at the time of Cro-Magnon man, about thirty thousand years ago, or in the Neanderthal period, more than fifty thousand years ago? Or could it have been half a million years ago? Or even – why not? – a million years ago?

What motives did prehistoric peoples have to begin the great adventure of counting? Were their concerns purely astronomical (to do with the phases of the moon, the eternal return of day and night, the cycle of the seasons, and so on)? Or did the requirements of communal living give the first impulse towards counting? In what way and after what period of time did people discover that the fingers of one hand and the toes of one foot represent the same concept? How did the need for calculation impose itself on their minds? Was there a chronological sequence in the discovery of the cardinal and ordinal aspects of the integers? In which period did the first attempts at oral numbering occur? Did an abstract conception of number precede articulated language? Did people count by gesture and material tokens before doing so through speech? Or was it the other way round? Does the idea of number come from experience of the world? Or did the idea of number act as a catalyst and make explicit what must have been present already as a latent idea in the minds of our most distant ancestors? And finally, is the concept of number the product of intense human thought, or is it the result of a long and slow evolution starting from a very concrete understanding of things?

These are all perfectly normal questions to ask, but most of the answers cannot be researched in a constructive way since there is no longer any trace of the thought-processes of early humans. The event, or, more probably, the sequence of events, has been lost in the depths of prehistoric time, and there are no archaeological remains to give us a clue.

However, archaeology was not necessarily the only approach to the problem. What other discipline might there be that would allow at least a stab at an answer? For instance, might psychology and ethnology not have some power to reconstitute the origins of number?

The Quest for Number? Or a quest for a wraith? That was the question. It was not easy to know which it was, but I had set out on it and was soon to conquer the whole world, from America to Egypt, from India to Mexico, from Peru to China, in my search for more and yet more numbers. But as I had no financial backer, I decided to be my own sponsor, doing odd jobs (delivery boy, chauffeur, waiter, night watchman) to keep body and soul together.

As an intellectual tourist I was able to visit the greatest museums in the world, in Cairo, Baghdad, Beijing, Mexico City, and London (the British Museum and the Science Museum); the Smithsonian in Washington, the Vatican Library in Rome, the libraries of major American universities (Yale, Columbia, Philadelphia), and of course the many Paris collections at the Musée Guimet, the Conservatoire des arts et métiers, the Louvre, and the Bibliothèque nationale. I also visited the ruins of Pompeii and Masada. And took a trip to the Upper Nile Valley to see Thebes, Luxor, Abu Simbel, Gizeh. Had a look at the Acropolis in Athens and the Forum in Rome. Pondered on time's stately march from the top of the Mayan pyramids at Quiriguá and Chichén Itzá. And from here and from there I gleaned precious information about past and present customs connected with the history of counting.

When I got back from these fascinating ethno-numerical and archaeo-arithmetical expeditions I buried myself in popularising and encyclopaedic articles, plunged into learned journals and works of erudition, and fired off thousands of questions to academic specialists in scores of different fields.

At the start, I did not get many replies. My would-be correspondents were dumbfounded by the banality of the topic.

There are of course vast numbers of oddballs forever pestering specialists with questions. But I had to persuade them that I was serious. It was essential for me to obtain their co-operation, since I needed to be kept up to date about new and recent discoveries in their fields, however apparently insignificant, and as an amateur I needed their help in avoiding misinterpretations. And since I was dealing with many specialists who were far outside the field of mathematics, I had not only to persuade them that I was an honest toiler in a respectable field, but also to get them to accept that "numbers" and "mathematics" are not quite the same thing. As we shall see . . .

All this work led me to two basic facts. First, a vast treasure-house of documentation on the history of numbers does actually exist. I owe a great deal to the work of previous scholars and mention it frequently throughout this book. Secondly, however, the articles and monographs in this store of knowledge each deal with only one specialism, are addressed to other experts in the same field, and are far from being complete or comprehensive accounts. There were also a few general works, to be sure, which I came across later, and which also gave me some help. But as they describe the state of knowledge at the time they were written, they had been long overtaken by later discoveries in archaeology, psychology, and ethnography.

No single work on numbers existed which covered the whole of the available field, from the history of civilisations and religions to the history of science, from prehistoric archaeology to linguistics and philology, from mythical and mathematical interpretation to ethnography, ranging over the five continents.

Indeed, how can one successfully sum up such heterogeneous material without losing important distinctions or falling into the trap of simplification? The history of numbers includes topics as widely divergent as the perception of number in mammals and birds, the arithmetical use of prehistoric notched bones, Indo-European and Semitic numbering systems, and number-techniques among so-called primitive populations in Australia, the Americas, and Africa. How can you catch in one single net things as different as finger-counting and digital computing? counting with beads and Amerindian or Polynesian knotted string? Pharaonic epigraphy and Babylonian baked clay tablets? How can you talk in the same way about Greek and Chinese arithmetic, astronomy and Mayan inscriptions, Indian poetry and mathematics, Arabic algebra and the mediaeval quadrivium? And all of that so as to obtain a coherent overall vision of the development through time and space of the defining invention of modern humanity, which is our present numbering system? And where do animals fit into what is already an enormously complex field? Not to mention human infants . . .

What I had set out to do was manifestly mad. The topic sat at the junction of all fields of knowledge and constituted an immense universe of human intellectual evolution. It covered a field so rich and huge that no single person could hope to grasp it alone.

Such a quest is by its nature unending. This book will occupy a modest place in a long line of outstanding treatises. It will not be the last of them, to be sure, for so many more things remain undiscovered or not yet understood. All the same, I think I have brought together practically everything of significance from what the number-based sciences, of the logical and historical kinds, have to teach us at the moment. Consequently, this is also probably the only book ever written that gives a more or less universal and comprehensive history of numbers and numerical calculation, set out in a logical and chronological way, and made accessible in plain language to the ordinary reader with no prior knowledge of mathematics.

And since research never stands still, I have been able to bring new solutions to some problems and to open up other, long-neglected areas of the universe of numbers. For example, in one of the chapters you will find a solution to the thorny problem of the decipherment of Elamite numbering, used nearly five thousand years ago in what is now Iran. I have also shown that Roman numbering, long thought to have been derived from the Greek system, was in fact a "prehistoric fossil", developed from the very ancient practice of notching. There are also some new contributions on Mesopotamian numbering and arithmetic, as well as a quite new way of looking at the fascinating and sensitive topic of how "our" numbers evolved from the unlikely conjunction of several great ideas. Similarly, the history of mechanical calculation culminating in the invention of the computer is entirely new.

A VERY LONG STORY

If you wanted to schematise the history of numbering systems, you could say that it fills the space between One and Zero, the two concepts which have become the symbols of modern technological society.

Nowadays we step with careless ease from Zero to One, so confident are we, thanks to computer scientists and our mathematical masters, that the Void always comes before the Unit. We never stop to think for a moment that in terms of time it is a huge step from the invention of the number "one", the first of all numbers even in the chronological sense, to the invention of the number "zero", the last major invention in the story of numbers. For in fact the whole history of humanity is spread out backwards between the time when it was realised that the void was "nothing" and the time when the sense of "oneness" first arose, as humans became aware of their individual solitude in the face of life and death, of the specificity of their species as distinct from other living beings, of the singularity of their selves as distinct from others, or of the difference of their sex as distinct from that of their partners.

But the story is neither abstract nor linear, as the history of mathematics is sometimes (and erroneously) imagined to be. Far from being an impeccable sequence of concepts each leading logically to the next, the history of numbers is the story of the needs and concerns of enormously diverse social groupings trying to count the days in the year, to make deals and bargains, to list their members, their marriages, their bereavements, their goods and flocks, their soldiers, their losses, and even their prisoners, trying also to record the date of the foundation of their cities or of one of their victories.

Goatherds and shepherds needed to know when they brought their flocks back from grazing that none had been lost; people who kept stocks of tools or arms or stood guard over food supplies for a community needed to know whether the complement of tools, arms or supplies had remained the same as when they last checked. Or again, communities with hostile neighbours must have been concerned to know whether, after each military foray, they still had the same number of soldiers, and, if not, how many they had lost in the fight. Communities that engaged in trading needed to be able to "reckon" so as to be able to buy or barter goods. For harvesting, and also in order to prepare in time for religious ceremonies, people needed to be able to count and to measure time, or at the very least to develop some practical means of managing in such circumstances.

In a word, the history of numbers is the story of humanity being led by the very nature of the things it learned to do to conceive of needs that could only be satisfied by "number reckoning". And to do that, everything and anything was put in service. The tools were approximate, concrete, and empirical ones before becoming abstract and sophisticated, originally imbued with strange mystical and mythological properties, becoming disembodied and generalisable only in the later stages.

Some communities were utilitarian and limited the aims of their counting systems to practical applications. Others saw themselves in the infinite and eternal elements, and used numbers to quantify the heavens and the earth, to express the lengths of the days, months and years since the creation of the universe, or at least from some date of origin whose meaning had subsequently been lost. And because they found that they needed to represent very large numbers, these kinds of communities did not just invent more symbols, but went down a path that led not only towards the fundamental rule of position, but also onto the track of a very abstract concept that we call "zero", whence comes the whole of mathematics.

THE FIRST STEPS

No one knows where or when the story began, but it was certainly a very long time ago. That was when people were unable to conceive of numbers as such, and therefore could not count. They were capable, at most, of the concepts of *one*, *two*, and *many*.

As a result of studies carried out on a wide range of beings, from crows to humans as diverse as infants, Pygmies, and the Amerindian inhabitants of Tierra del Fuego, psychologists and ethnologists have been able to establish the absolute zero of human number-perception. Like some of the higher animals, the human adult with no training at all (for example, learning to recognise the 5 or the 6 at cards by sight, through sheer practice) has direct and immediate perception of the numbers 1 to 4 only. Beyond that level, people have to learn to count. To do that they need to develop, firstly, advanced number-manipulating skills, then, for the purposes of memorisation and of communication, they need to develop a linguistic instrument (the names of the numbers), and, finally, and much later on, they need to devise a scheme for writing numbers down.

However, you do not have to "count" the way we do if what you want to do is to find the date of a ceremony, or to make sure that the sheep and the goats that set off to graze have all come back to the byre. Even in the complete absence of the requisite words, of sufficient memory, and of the abstract concepts of number, there are all sorts of effective substitute devices for these kinds of operation. Various present-day populations in Oceania, America, Asia, and Africa whose languages contain only the words for *one*, *two*, and *many*, but who nonetheless understand one-for-one parities perfectly well, use notches on bones or wooden sticks to keep a tally. Other populations use piles or lines of pebbles, shells, knucklebones, or sticks. Still others tick things off by the parts of their body (fingers, toes, elbows and knees, eyes, nose, mouth, ears, breasts, and chest).

THE EARLIEST COUNTING MACHINES

Early humanity used more or less whatever came to hand to manage in a quantitative as well as a qualitative universe. Nature itself offered every cardinal model possible: birds with two wings, the three parts of a clover-leaf, four-legged animals, and five-fingered hands . . . But as everyone began counting by using their ten fingers, most of the numbering systems that were invented used base 10. All the same, some groups chose base 12. The Mayans, Aztecs, Celts, and Basques, looked down at their feet and realised that their toes could be counted like fingers, so they chose base 20. The Sumerians and Babylonians, however, chose to count on base 60, for reasons that remain mysterious. That is where our present division of the hour into 60 minutes of 60 seconds comes from, as does the division of a circle into 360 degrees, each of 60 minutes divided into 60 seconds.

The very oldest counting tools that archaeologists have yet dug up are the numerous animal bones found in western Europe and marked with one or more sets of notches. These tally sticks are between twenty thousand and thirty-five thousand years old.

The people using these bones were probably fearsome hunters, and, for each kill, they would score another mark onto the tally stick. Separate counting bones might have been used for different animals – one tally for bears, another for bison, another for wolves, and so on.

They had also invented the first elements of accounting, since what they were actually doing was writing numbers in the simplest notation known.

The method may seem primitive, but it turned out to be remarkably robust, and is probably the oldest human invention (apart from fire) still in use today. Various tallies found on cave walls next to animal paintings leave us in little real doubt that we are dealing with an animal-counting device. Modern practice is no different. Since time immemorial, Alpine shepherds in Austria and Hungary, just like Celtic, Tuscan, and Dalmatian herdsmen, have checked off their animals by scoring vertical bars, Vs and Xs on a piece of wood, and that is still how they do it today. In the eighteenth century, the same "five-barred gate" was used for the shelf marks of parliamentary papers at the British House of Commons Library; it was used in Tsarist Russia and in Scandinavia and the German-speaking countries for recording loans and for calendrical accounts; whereas in rural France at that time, notched sticks did all that present-day account books and contracts do, and in the open markets of French towns they served as credit "slates". Barely twenty years ago a village baker in Burgundy made notches in pieces of wood when he needed to tot up the numbers of loaves each of his customers had taken on credit. And in nineteenth-century Indo-China, tally sticks were used as credit instruments, but also as signs of exclusion and to prevent contact with cholera victims. Finally, in Switzerland, we find notched sticks used, as elsewhere, for credit reckonings, but also for contracts, for milk deliveries, and for recording the amounts of water allocated to different grazing meadows.

The long-lasting and continuing currency of the tally system is all the more surprising for being itself the source of the Roman numbering system, which we also still use alongside or in place of Arabic numerals.

The second concrete counting tool, the hand, is of course even older. Every population on earth has used it at one stage or another. In various places in Auvergne (France), in parts of China, India, Turkey, and the former Soviet Union, people still do multiplication sums with their fingers, as the numbers are called out, and without any other tool or device. Using joints and knuckles increases the possible range, and it allowed the Ancient Egyptians, the Romans, the Arabs and the Persians (not forgetting Western Christians in the Middle Ages) to represent concretely all the numbers from 1 to 9,999. An even more ingenious variety of finger-reckoning allowed the Chinese to count to 100,000 on one hand, and to one million using both hands!

But the story of numbers can be told in other ways too. In places as far apart as Peru, Bolivia, West Africa, Hawaii, the Caroline Islands, and Ryû-Kyû, off the Japanese coast, you can find knotted string used to represent numbers. It was with such a device that the Incas sorted the archives of their very effective administration.

A third system has a far from negligible role in the history of arithmetic – the use of pebbles, which really underlies the beginning of calculation. The pebble-method is also the direct ancestor of the abacus, a device still in wide use in China, Japan and Eastern Europe. But it is the very word *calculation* that sends us back most firmly to the pebble-method: for in Latin the word for pebble is *calculus*.

THE FIRST NUMBERS IN HISTORY

The pebble-method actually formed the basis for the first written numbering system in recorded history. One day, in the fourth millennium BCE, in Elam, located in present-day Iran towards the Persian Gulf, accountants had the idea of using moulded, unbaked clay tokens in the place of ordinary or natural pebbles. The tokens of various shapes and sizes were given conventional values, each different type representing a unit of one order of magnitude within a numbering system: a stick shape for 1, a pellet for 10, a ball for 100, and so on. The idea must have been in the air for a long time, for at about the same period a similarly clay-based civilisation in Sumer, in lower Mesopotamia, invented an identical system. But since the Sumerians counted to base 60 (sexagesimal reckoning), their system was slightly different: a small clay cone stood for 1, a pellet stood for 10, a large cone for 60, a large perforated cone stood for 600, a ball meant 3,600, and so on.

These civilisations were in a phase of rapid expansion but remained exclusively oral, that is to say without writing. They relied on the rather limited potential of human memory. But the accounting system that was developed from the principles just explained turned out to be very serviceable. In the first development, the idea arose of enclosing the tokens in a spherical clay case. This allowed the system not only to serve for actual arithmetical operations, but also for keeping a record of inventories and transactions of all kinds. If a check on past dealings was needed, the clay cases could be broken open. But the second development was even more pregnant. The idea was to symbolise on the outside of the clay case the objects that were enclosed within it: one notch on the case signified that there was one small cone inside, a pellet was symbolised by a small circular perforation, a large cone by a thick notch, a ball by a circle, and so on. Which is how the oldest numbers in history, the Sumerian numerals, came into being, around 3200 BCE.

This story is obviously related to the origins of writing, but it must not be confused with it entirely. Writing serves not only to give a visual representation to thought and a physical form to memory (a need felt by all advanced societies), but above all to record articulated speech.

THE COMMON STRUCTURE OF THE HUMAN MIND

It is extraordinary to see how peoples very distant from each other in time and space used similar methods to reach identical results.

All societies learned to number their own bodies and to count on their fingers; and the use of pebbles, shells and sticks is absolutely universal. So the fact that the use of knotted string occurs in China, in Pacific island communities, in West Africa, and in Amerindian civilisations does not require us to speculate about migrations or long-distance travellers in prehistory. The making of notches to represent number is just as widespread in historical and geographical terms. Since the marking of bone and wood has the same physical requirements and limitations wherever it is done, it is no surprise that the same kinds of lines, Vs and Xs are to be seen on armbones and pieces of wood found in places as far apart as Europe, Asia, Africa, Oceania and the Americas. That is also why these marks crop up in virtually identical form in civilisations as varied as those of the Romans, the Chinese, the Khâs Boloven of Indo-China, the Zuñi Indians of New Mexico, and amongst contemporary Dalmatian and Celtic herdsmen. It is therefore not at all surprising that some numbers have almost always been represented by the same figure: 1, for instance, is represented almost universally by a single vertical line; 5 is also very frequently, though slightly less universally, figured by a kind of V in one orientation or another, and 10 by a kind of X or by a horizontal bar.

Similarly, the Ancient Egyptians, the Hittites, the Greeks, and the Aztecs worked out written numbering systems that were structurally identical, even if their respective base numbers and figurations varied considerably. Likewise the common system of Sumerian, Roman, Attic, and South Arabian numbering. Several family groupings of the same kind can be found in other sets of unrelated cultures. There is no need to hypothesise actual contact between the cultures in order to explain the similarities between their numbering systems.

So it would seem that human beings possess, in all places and at all times, a permanent capacity to repeat an invention or discovery already made elsewhere, provided only that the society or individual involved encounters cultural, social, and psychological conditions similar to those that prevailed when the invention was first made.

This is what explains why in modern science, the same discovery is sometimes made at almost the same time by two different scientists working in complete isolation from each other. Famous examples of such coincidences of invention include the simultaneous development of analytical geometry by Descartes and Fermat, of differential calculus by Newton and Leibnitz, of the physical laws of gasses by Boyle and Mariotte, and of the principles of thermodynamics by Joule, Mayer, and Sadi Carnot.

NUMBERS AND LETTERS

Ever since the invention of alphabetic writing by the Phoenicians (or at least, by a northwestern Semitic people) in the second millennium BCE, letters have been used for numbers. The simplicity and ingenuity of the alphabetic system led to its becoming the most widespread form of writing, and the Phoenician scheme is at the root of nearly every alphabet in the world today, from Hebrew to Arabic, from Berber to Hindu, and of course Greek, which is the basis of our present (Latin) lettering.

Given their alphabets, the Greeks, the Jews, the Arabs and many other peoples thought of writing numbers by using letters. The system consists of attributing numerical values from 1 to 9, then in tens from 10 to 90, then in hundreds, etc., to the letters in their original Phoenician order (an order which has remained remarkably stable over the millennia).

Number-expressions constructed in this way worked as simple accumulations of the numerical values of the individual letters. The mathematicians of Ancient Greece rationalised their use of letter-numbers within a decimal system, and, by adding diacritic signs to the base numbers, became able to express numbers to several powers of 10.

In poetry and literature, however, and especially in the domains of magic, mysticism, and divination, it was the sum of the number-values of the letters in a word that mattered.

In these circumstances, every word acquired a number-value, and conversely, every number was "loaded" with the symbolic value of one or more words that it spelled. That is why the number 26 is a divine number in Jewish lore, since it is the sum of the number-values of the letter that spell YAHWEH, the name of God:

ה ו ה י = 5 + 6 + 5 + 10

The Jews, Greeks, Romans, Arabs (and as a result, Persians and Muslim Turks) pursued these kinds of speculation, which have very ancient origins: Babylonian writings of the second millennium BCE attribute a numerical value to each of the main gods: 60 was associated with Anu, god of the sky; 50 with Enlíl, god of the earth; 40 with Ea, god of water, and so forth.

The device also allowed poets like Leonidas of Alexandria to compose quite special kinds of work. It is also the basis for the art of the chronogram (verses that express a date simultaneously in words and in numbers) that can be found amongst the poets and stone-carvers of North Africa, Turkey, and Iran.

From ancient times to the present, the device has given a rich field to cabbalists, Gnostics, magicians, soothsayers, and mystics of every hue, and innumerable speculations, interpretations, calculations and predictions have been built on letter-number equivalences. The Gnostics, for example, thought they could work out the "formula" and thus the true name of God, which would enable them to penetrate all the secrets of the divine. Several religious sects are based on beliefs of this kind (such as the Hurufi or "Lettrists" of Islam) and they still have many followers, some of them in Europe.

The Greeks and Jews who first established a number-coded alphabet certainly could not have imagined that fifteen hundred or two thousand years later a Catholic theologian called Petrus Bungus would churn out a seven-hundred page numerological treatise "proving" (subject to a few spelling improvements!) that the name of Martin Luther added up to 666. It was a proof that the "isopsephic" initiates knew how to read, since according to St John the Apostle, 666 was the number of the "Beast of the Apocalypse", that is to say the Antichrist. Bungus was neither the first nor the last to make use of these methods. In the late Roman Empire, Christians tried to make Nero's name come to 666; during World War II, would-be numerological prophets managed to "prove" that Hitler was the real "Beast of the Apocalypse". A discovery that many had already made without the help of numbers.

THE HISTORY OF A GREAT INVENTION

Logic was not the guiding light of the history of number-systems. They were invented and developed in response to the concerns of accountants, first of all, but also of priests, astronomers, and astrologers, and only in the last instance in response to the needs of mathematicians. The social categories dominant in this story are notoriously conservative, and they probably acted as a brake on the development and above all on the accessibility of numbering systems. After all, knowledge (however rudimentary it may now appear) gives its holders power and privilege; it must have seemed dangerous, if not irreligious, to share it with others.

There were also other reasons for the slow and fragmentary development of numbers. Whereas fundamental scientific research is pursued in terms of scientists' own criteria, inventions and discoveries only get developed and adopted if they correspond to a perceived social need in a civilisation. Many scientific advances are ignored if there is, as people say, no "call" for them.

The stages of mathematical thought make a fascinating story. Most peoples throughout history failed to discover the rule of position, which was discovered in fact only four times in the history of the world. (The rule of position is the principle of a numbering system in which a 9, let's say, has a different magnitude depending on whether it comes in first, second, third . . . position in a numerical expression.) The first discovery of this essential tool of mathematics was made in Babylon in the second millennium BCE. It was then rediscovered by Chinese arithmeticians at around the start of the Common Era. In the third to fifth centuries CE, Mayan astronomers reinvented it, and in the fifth century CE it was rediscovered for the last time, in India.

Obviously, no civilisation outside of these four ever felt the need to invent zero; but as soon as the rule of position became the basis for a numbering system, a zero was needed. All the same, only three of the four (the Babylonians, the Mayans and the Indians) managed to develop this final abstraction of number: the Chinese only acquired it through Indian influences. However, the Babylonian and Mayan zeros were not conceived of as numbers, and only the Indian zero had roughly the same potential as the one we use nowadays. That is because it is indeed the Indian zero, transmitted to us through the Arabs together with the number-symbols that we call Arabic numerals and which are in reality Indian numerals, with their appearance altered somewhat by time, use and travel.

Our knowledge of the history of numbers is of course only fragmentary, but all the pieces converge inexorably towards the system that we now use and which in recent times has conquered the whole planet.

COMPUTATION, FIGURES, AND NUMBERS

Arithmetic has a history that is by no means limited to the history of the figures we use to represent numbers. In this history of computation, figures arose quite late on; and they constitute only one of many possible ways of representing number-concepts. The history of numbers ran parallel to the history of computation, became part of it only when modern written arithmetic was invented, and then separated out again with the development of modern calculating machines.

Numbers have become so integrated into our way of thinking that they often seem to be a basic, innate characteristic of human beings, like walking or speaking. But that is not so. Numbers belong to human culture, not nature, and therefore have their own long history. For Plato, numbers were "the highest degree of knowledge" and constituted the essence of outer and inner harmony. The same idea was taken up in the Middle Ages by Nicholas Cusanus, for whom "numbers are the best means of approaching divine truths". These views all go back to Pythagoras, for whom "numbers alone allow us to grasp the true nature of the universe".

In truth, though, it is not numbers that govern the universe. Rather, there are physical properties in the world which can be expressed in abstract terms through numbers. Numbers do not come from things themselves, but from the mind that studies things. Which is why the history of numbers is a profoundly human part of human history.

IN CONCLUSION

Once a person's curiosity, on any subject, is aroused it is surprising just how far it may lead him in pursuit of its object, how readily it overcomes every obstacle. In my own case my curiosity about, or rather my absolute fascination with, numbers has been well served by a number of assets with which I set out: a Moroccan by birth, a Jew by cultural heritage, I have been afforded a more immediate access to the study of the work of Arab and Hebrew mathematicians than I might have obtained as a born European. I could harmonise within myself the mind-set of Eastern metaphysics with the Cartesian logic of the West. And I was able to identify the basic rules of a highly complex system. Moreover I possessed a sufficient aptitude for drawing to enable me to make simple illustrations to help clarify my text. I hope that the reader will recognise in this History that numbers, far from being tedious and dry, are charged with poetry, are the very vehicle for traditional myths and legends – and the finest witness to the cultural unity of the human race.